

Test Booklet Series



Test Booklet
(Mathematics)

Test Booklet No.

Name of Applicant

Answer Sheet No.

Applicant ID/Roll No. :

Signature of Applicant :

Date of Examination :

Signature of the Invigilator(s)

Time of Examination :

1.

2.

Duration : 2 Hour]

[Maximum Marks : 100

IMPORTANT INSTRUCTIONS

- (i) The question paper is in the form of Test-Booklet containing **100 (Hundred)** questions. All questions are compulsory. Each question carries four answers marked (A), (B), (C) and (D), out of which only one is correct. Choose the correct option or the most appropriate option.
- (ii) On receipt of the Test-Booklet (Question Paper), the candidate should immediately check it and ensure that it contains all the pages, i.e., **100** questions. Discrepancy, if any, should be reported by the candidate to the invigilator immediately after receiving the Test-Booklet.
- (iii) A separate Answer-Sheet is provided with the Test-Booklet/Question Paper. On this sheet there are **100** rows containing four circles each. One row pertains to one question.
- (iv) The candidate should write his/her Application ID/Roll number at the places provided on the cover page of the Test-Booklet/Question Paper and on the Answer-Sheet and **NOWHERE ELSE**.
- (v) No second Test-Booklet/Question Paper and Answer-Sheet will be given to a candidate. The candidates are advised to be careful in handling it and writing the answer on the Answer-Sheet.
- (vi) For every correct answer of the question **One (1) mark will be awarded**.
- (vii) Marking shall be done only on the basis of answers responded on the Answer-Sheet.
- (viii) To mark the answer on the Answer-Sheet, candidate should darken the appropriate circle in the row of each question with Blue or Black pen.
- (ix) For each question only **one** circle should be **darkened** as a mark of the answer adopted by the candidate. If more than one circle for the question are found darkened or with one black circle any other circle carries any mark, the answer will be treated as incorrect.
- (x) The candidates should not remove any paper from the Test-Booklet/Question Paper. Attempting to remove any paper shall be liable to be punished for use of unfair means.
- (xi) Rough work may be done on the blank space provided in the Test-Booklet/Question Paper only.
- (xii) *Mobile phones (even in Switch-off mode) and such other communication/programmable devices are not allowed inside the examination hall.*
- (xiii) No candidate shall be permitted to leave the examination hall before the expiry of the time.

DO NOT OPEN THIS QUESTION BOOKLET UNTIL ASKED TO DO SO.

Mathematics

[P.T.O.

13 / 1

PART-A
(Research Methodology)

1. The main objective of fundamental/basic research is:
(A) To solve an immediate practical problem
(B) To discover new facts and develop theories
(C) To evaluate existing social programs
(D) To study historical events

2. The scientific method of research is characterized by:
(A) Subjectivity and intuition
(B) Empiricism, objectivity, and replicability
(C) Relying solely on authority
(D) Lack of logical reasoning

3. While selecting a research topic, which of the following is the LEAST important consideration?
(A) Feasibility of the study
(B) Researcher's interest
(C) Availability of funds and time
(D) Personal biases of the researcher

4. The ethical principle of "Informed Consent" implies that:
(A) Participants must be forced to participate for the greater good
(B) Participants must be fully aware of the research procedures and agree voluntarily
(C) Researchers can hide the true purpose of the study to avoid bias
(D) Data can be shared publicly without participant knowledge

5. Fabrication, falsification, and plagiarism in research are collectively known as:
(A) Research misconduct
(B) Sampling error
(C) Ethical compliance
(D) Research methodology

6. Which research design is primarily concerned with describing the characteristics of a population or phenomenon?
(A) Exploratory research design
(B) Descriptive research design
(C) Experimental research design
(D) Diagnostic research design

7. In an experimental research design, the variable that is intentionally manipulated by the researcher is the:
(A) Dependent variable
(B) Extraneous variable
(C) Independent variable
(D) Control variable

8. Research conducted to find out the causes of an event that has already happened, without manipulating variables, is known as:
- (A) Action research (B) Ex-post facto research
(C) Applied research (D) Pure research
9. The "Control Group" in an experimental design is used to:
- (A) Introduce the experimental treatment
(B) Serve as a baseline for comparison with the experimental group
(C) Eliminate the dependent variable
(D) Increase the sample size
10. Deductive reasoning in the scientific method moves from:
- (A) Specific observations to general theories
(B) General theories to specific hypotheses
(C) Empirical data to conceptualization
(D) Null hypothesis to alternative hypothesis
11. Internal validity in an experimental design refers to:
- (A) The generalizability of the findings to the real world
(B) The certainty that the independent variable caused the changes in the dependent variable
(C) The consistency of the measuring instrument
(D) The representativeness of the sample
12. If every member of the population has an equal and independent chance of being selected, the sampling technique is called:
- (A) Purposive sampling (B) Convenience sampling
(C) Simple random sampling (D) Quota sampling
13. When the population is heterogeneous with distinct subgroups (strata), the most appropriate probability sampling technique is:
- (A) Simple random sampling (B) Stratified random sampling
(C) Cluster sampling (D) Snowball sampling

- 14.** In cluster sampling, the sampling units selected are:
- (A) Individual elements
 - (B) Natural groupings or categories (e.g., districts, schools)
 - (C) Homogeneous subgroups
 - (D) Volunteers
- 15.** Snowball sampling is specifically used when:
- (A) The population size is very large
 - (B) The sampling frame is readily available
 - (C) The population is hidden, rare, or difficult to access
 - (D) The researcher needs a perfectly representative sample
- 16.** The difference between a sample statistic and the true population parameter is termed as:
- (A) Systematic error
 - (B) Sampling error
 - (C) Measurement error
 - (D) Non-sampling error
- 17.** Which measurement scale classifies data into mutually exclusive categories without any order or ranking?
- (A) Ordinal scale
 - (B) Interval scale
 - (C) Nominal scale
 - (D) Ratio scale
- 18.** A Likert scale (Strongly Disagree to Strongly Agree) is technically an example of:
- (A) Nominal scale
 - (B) Ordinal scale
 - (C) Interval scale
 - (D) Ratio scale
- 19.** Which scaling technique asks respondents to rate an object on a series of bipolar adjectives (e.g., Good-Bad, Fast-Slow)?
- (A) Likert scale
 - (B) Guttman scale
 - (C) Semantic Differential scale
 - (D) Thurstone scale

- 20.** The key difference between a questionnaire and a schedule is:
- (A) A questionnaire is filled by the enumerator, a schedule is filled by the respondent
 - (B) A questionnaire is filled by the respondent, a schedule is filled by the enumerator/ interviewer
 - (C) A schedule is mailed, a questionnaire is given face-to-face
 - (D) There is no difference between them
- 21.** Data collected by the researcher themselves for the specific purpose of the current study is called:
- (A) Secondary data
 - (B) Primary data
 - (C) Official data
 - (D) Documentary data
- 22.** The process of checking the raw data to ensure completeness, accuracy, and consistency before analysis is called:
- (A) Coding
 - (B) Editing
 - (C) Tabulation
 - (D) Classification
- 23.** Assigning numerical values or symbols to the responses for computational purposes is known as:
- (A) Editing
 - (B) Coding
 - (C) Transcription
 - (D) Tabulation
- 24.** A histogram is used to graphically represent the distribution of:
- (A) Discrete/categorical data
 - (B) Continuous data grouped into class intervals
 - (C) Qualitative characteristics
 - (D) Proportions of a whole
- 25.** A pie chart is most effective for representing:
- (A) Trends over time
 - (B) The relationship between two continuous variables
 - (C) The percentage breakdown of a whole
 - (D) Frequency distribution of continuous data

26. A hypothesis that states there is no significant difference or relationship is called:
- (A) Directional hypothesis (B) Non-directional hypothesis
(C) Null hypothesis (D) Alternative hypothesis
27. "Students who study using method A will score significantly higher than those who study using method B." This is an example of:
- (A) Null hypothesis
(B) Directional alternative hypothesis
(C) Non-directional alternative hypothesis
(D) Ex-post facto hypothesis
28. A Type I error in hypothesis testing occurs when:
- (A) We fail to reject a true null hypothesis
(B) We reject a true null hypothesis
(C) We fail to reject a false null hypothesis
(D) We reject a false null hypothesis
29. The probability of committing a Type I error is denoted by:
- (A) Beta (β) (B) Alpha (α)
(C) P-value (D) Standard error
30. A statistical test that makes assumptions about the population parameters (like normal distribution, homogeneity of variance) is called:
- (A) Non-parametric test (B) Parametric test
(C) Chi-square test (D) Distribution-free test
31. The Chi-square (χ^2) test is primarily used to test:
- (A) The difference between means of two groups
(B) The association between two categorical variables
(C) The significance of regression coefficients
(D) The variance within a single group

32. In a Chi-square test of independence, if the calculated value of χ^2 is greater than the critical/ table value at the 0.05 level, we:
- (A) Accept the null hypothesis
 - (B) Reject the null hypothesis
 - (C) Conclude the variables are independent
 - (D) Conclude the sample is biased
33. The formula for calculating expected frequency (E) in a Chi-square contingency table cell is:
- (A) (Row Total $\times$ Column Total) / Grand Total
 - (B) (Row Total + Column Total) / Grand Total
 - (C) Grand Total / (Row Total $\times$ Column Total)
 - (D) Row Total / Column Total
34. Analysis of Variance (ANOVA) is used instead of multiple t-tests to compare the means of three or more groups because:
- (A) ANOVA is easier to calculate
 - (B) Multiple t-tests increase the risk of Type I error
 - (C) ANOVA does not require normal distribution
 - (D) ANOVA uses nominal data
35. In a one-way ANOVA, the "one way" refers to:
- (A) One dependent variable and one independent variable with multiple levels
 - (B) One independent variable and multiple dependent variables
 - (C) Only one sample being tested
 - (D) One-tailed test of significance
36. The F-ratio in One-way ANOVA is calculated as:
- (A) Within-group variance / Between-group variance
 - (B) Between-group variance / Within-group variance
 - (C) Total variance / Residual variance
 - (D) Mean square error / Mean square total
37. The degrees of freedom for "Between Groups" in a One-Way ANOVA with 'k' groups is:
- (A) $N - 1$
 - (B) $N - k$
 - (C) $k - 1$
 - (D) $(N - 1) - (k - 1)$

38. Which of the following is a non-parametric alternative to One-way ANOVA?
- (A) Kruskal-Wallis test (B) Mann-Whitney U test
(C) Wilcoxon signed-rank test (D) Chi-square goodness of fit
39. A multivariate statistical technique used to reduce a large number of interrelated variables to a smaller set of unobservable factors is:
- (A) Cluster Analysis (B) Factor Analysis
(C) Discriminant Analysis (D) Conjoint Analysis
40. In Factor Analysis, the proportion of variance in a specific variable explained by the common factors is called:
- (A) Communality (B) Eigenvalue
(C) Factor loading (D) Uniqueness
41. Which concept in Factor Analysis represents the total variance explained by a specific factor across all variables?
- (A) Communality (B) Eigenvalue
(C) Factor score (D) Rotation
42. Multiple Regression is used to:
- (A) Classify objects into groups
(B) Predict the value of a dependent variable based on two or more independent variables
(C) Reduce the number of variables
(D) Determine the association between two categorical variables
43. In Multiple Regression, the coefficient of multiple determination (R^2) represents:
- (A) The correlation between two independent variables
(B) The proportion of variance in the dependent variable explained by all independent variables
(C) The regression coefficient of the strongest predictor
(D) The standard error of the estimate
44. Pearson's correlation coefficient (r) measures:
- (A) The causal relationship between two variables
(B) The linear association between two continuous variables
(C) The association between two categorical variables
(D) The variance of a single variable

- 45.** If the Pearson correlation coefficient between X and Y is -0.85 , it indicates:
- (A) A weak positive relationship (B) A strong positive relationship
(C) A strong negative relationship (D) No relationship
- 46.** Which multivariate technique is used to group objects or individuals so that objects in the same group are highly similar to each other but different from objects in other groups?
- (A) Discriminant Analysis (B) Factor Analysis
(C) Cluster Analysis (D) Conjoint Analysis
- 47.** The main difference between Cluster Analysis and Discriminant Analysis is that:
- (A) Cluster analysis requires predefined groups, discriminant analysis does not
(B) Discriminant analysis requires predefined groups, cluster analysis does not
(C) Cluster analysis uses metric data, discriminant analysis uses non-metric
(D) Discriminant analysis is for data reduction, cluster analysis is for prediction
- 48.** Conjoint Analysis is primarily used in marketing research to:
- (A) Determine the optimal number of clusters
(B) Measure how consumers value different features of a product by analyzing trade-offs
(C) Reduce the dimensions of product features
(D) Classify customers into predefined demographic groups
- 49.** The standard format for writing a quantitative research report is generally known by the acronym:
- (A) IMRAD (Introduction, Method, Results, And Discussion)
(B) PEST (Political, Economic, Social, Technological)
(C) SWOT (Strengths, Weaknesses, Opportunities, Threats)
(D) APA (American Psychological Association)
- 50.** Which section of the research report contains a brief summary of the entire study, including the problem, methodology, major findings, and conclusions?
- (A) Introduction (B) Abstract/Executive Summary
(C) Literature Review (D) Discussion

PART-B
(Mathematics)

51. Which of the following function is uniformly continuous on the interval $(0, 1)$?

(A) $\sin\left(\frac{1}{x}\right)$

(B) $x \sin\left(\frac{1}{x}\right)$

(C) $\frac{1}{x}$

(D) None of these.

52. The series $\frac{1}{2} + \frac{1 \cdot 3}{2 \cdot 5} + \frac{1 \cdot 3 \cdot 5}{2 \cdot 5 \cdot 8} + \dots$ is

(A) Convergent

(B) Divergent

(C) Oscillatory

(D) None of these.

53. If E be a bounded closed set of real numbers then each open cover of E has a finite subcover. This represents

(A) Archimedean property

(B) Bolzana Weierstrass Theorem

(C) Heine–Borel Theorem

(D) Denseness Property.

54. For the function $f(x) = |x|$. Lagrange's mean value theorem does not hold in the interval

(A) $[-1, 0]$

(B) $\left[0, \frac{1}{2}\right]$

(C) $[0, 1]$

(D) $[-1, 1]$.

55. The infimum and the supremum of the set $\left\{ \frac{(-1)^n}{n} : n \in \mathbb{N} \right\}$ are respectively

(A) $1, -\frac{1}{2}$

(B) $-1, \frac{1}{2}$

(C) $-1, 0$

(D) None of these.

56. If a function f defined on $[0, 1]$ as

$$f(x) = \begin{cases} \sin\left(\frac{1}{x}\right), & \text{if } x \text{ is irrational} \\ 0, & \text{otherwise} \end{cases}$$

then

(A) f is not bounded

(B) f is R-integrable

(C) f is not R-integrable since f is not bounded

(D) f is not R-integrable since lower and upper integrals of f are unequal.

57. Consider the following improper integrals :

$$I_1 = \int_0^{\infty} \frac{\sin^2 x}{x^2} dx \quad \text{and} \quad I_2 = \int_1^{\infty} \frac{x^3}{(1+x)^5} dx, \quad \text{then}$$

(A) Both are convergent

(B) Both are divergent

(C) I_1 converges but not I_2

(D) I_2 converges but not I_1 .

58. The metric space $(\mathbb{R}, d)$, where d is a usual metric, is

(A) Compact

(B) Compact and connected

(C) Disconnected

(D) Connected but not compact.

59. The dimension of the subspace W of $\mathbb{R}^4$ generated by $\{(3, 8, -3, -5), (1, -2, 5, -3), (2, 3, 1, -4)\}$ is

- (A) 1 (B) 3
(C) 2 (D) None of these.

60. Which of the following is a linear transformation?

- (A) $T(x, y) = (1 + x, y)$ for all $(x, y) \in \mathbb{R}^2$
(B) $T(x, y) = (x, x + y)$ for all $(x, y) \in \mathbb{R}^2$
(C) $T(x, y) = (x^2, y)$ for all $(x, y) \in \mathbb{R}^2$
(D) None of these.

61. Consider the matrix $A = \begin{bmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{bmatrix}$, then

- (A) The matrix A has both positive and negative real eigen-values
(B) The matrix A has no real eigen-values
(C) All the real eigen-values of the matrix A are positive
(D) All the real eigen-values of the matrix A are negative.

62. The linear transformation $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ such that $T(X) = AX$, where $A = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 1 \\ -1 & 1 & 2 \end{bmatrix}$ is

- (A) $T(x_1, x_2, x_3) = (x_1 + x_3, 2x_1 + x_2 + x_3, -x_1 + x_2 + 2x_3)$
(B) $T(x_1, x_2, x_3) = (x_1 - x_3, 2x_1 - x_2 + x_3, x_1 - x_2 - 2x_3)$
(C) $T(x_1, x_2, x_3) = (x_1 + x_3, 2x_1 + x_2 - x_3, -x_1 + x_2 - 2x_3)$
(D) None of these.

63. The quadratic form $6x_1^2 + 3x_2^2 + 3x_3^2 - 4x_1x_2 - 2x_2x_3 + 4x_3x_1$ is
- (A) Positive definite
- (B) Negative definite
- (C) Non-negative definite but not positive definite
- (D) Neither negative definite nor positive definite.
64. Let C^3 be a complex inner product space. If the vectors $u_1 = (1, 2i, i)$, $u_2 = (0, 1 + i, 1)$, $u_3 = (2, 1 - i, i) \in C^3$, then the vector orthogonal to both u_1 and u_3 is
- (A) $(-3 + i, -i, 1 - 5i)$
- (B) $(-3 + i, -i, 1 + 5i)$
- (C) $(3 + i, -i, 1 + 5i)$
- (D) None of these.
65. If $V = \frac{x-y}{x^2+y^2}$ is the imaginary part of an analytic function $f(z)$, then
- (A) $f(z) = e^z + c$
- (B) $f(z) = \log z + c$
- (C) $f(z) = \frac{1+i}{z} + c$
- (D) None of these.
66. The residue of the function $f(z) = \frac{1}{(z^2+1)^3}$ at $z = i$, is
- (A) $\frac{3}{16i}$
- (B) $\frac{3}{2i}$
- (C) $\frac{4}{3i}$
- (D) None of these.

67. The value of the integral $\int_C \frac{e^{-z}}{z^2} dz$, where C is a unit circle about the origin, described in positive sense, is equal to
- (A) 0 (B) πi
- (C) $-2\pi i$ (D) $-\frac{\pi}{2}i$.
68. The fixed points of the Mobius transformation $\omega = \frac{(2+i)z-2}{z+i}$ are
- (A) $i, -i$ (B) $-1, 1$
- (C) $0, 1$ (D) $1+i, 1-i$.
69. If 9 colours are used to paint 100 houses, then atleast houses will be of the same colour.
- (A) 18 (B) 15
- (C) 12 (D) 10.
70. The congruence $15x \equiv 12 \pmod{21}$ has
- (A) 2 solution (B) 3 solution
- (C) 5 solution (D) No solution.
71. Let $G = \{0, 1, 2, 3, 4, 5, 6, 7\}$ be a group under the binary operation “addition modulo 8”, then the order of element 5, is
- (A) 1 (B) 2
- (C) 4 (D) 8.

72. Let G be a group of order 20449. Then
- (A) G has only one Sylow-11 subgroup
 - (B) G has only two Sylow-11 subgroups
 - (C) G has only four Sylow-11 subgroups
 - (D) None of these.
73. If R is a ring such that $x^2 = x$ for all $x \in R$, then R is a commutative ring of characteristic
- (A) 3
 - (B) 2
 - (C) 1
 - (D) None of these.
74. Which one of the following polynomial is irreducible over the field $\mathbb{Q}$ of rational numbers?
- (A) $x^3 + 3x + 1$
 - (B) $2x^5 + 15x^3 + 3x + 6$
 - (C) $8x^3 - 6x - 1$
 - (D) All of these.
75. The degree of splitting field of $x^4 - x^2 - 2$ over the field $\mathbb{Q}$ of rational numbers, is
- (A) 4
 - (B) 3
 - (C) 2
 - (D) None of these.
76. Which one of the following topological spaces is not compact?
- (A) Indiscrete topological space
 - (B) A topological space with cofinite topology
 - (C) Infinite discrete topological space
 - (D) None of these.

77. Let X be a topological space and U be a proper dense open subset of X . Then which of the following statement is true?
- (A) If X is compact, then U is compact
 (B) If X is connected, then U is connected
 (C) If $X \setminus U$ is compact, then X is compact
 (D) If X is compact, then $X \setminus U$ is compact.
78. The critical point $(0, 0)$ for the system $\frac{dx}{dt} = 2x - 4y$; $\frac{dy}{dt} = 2x - 2y$ is a
- (A) Stable spiral point (B) Unstable spiral point
 (C) Stable centre point (D) Stable node.
79. The particular integral of $y''' + 4y' = \sin 2x$ is
- (A) $\frac{x}{8} \sin 2x$ (B) $-\frac{x}{8} \sin 2x$
 (C) $\frac{x}{4} \sin 2x$ (D) $-\frac{x}{4} \sin 2x$.
80. Let n be non-negative integer. The eigen-values of Sturm-Liouville problem $y'' + \lambda y = 0$ with Boundary conditions $y(0) = y(2\pi)$, $y'(0) = y'(2\pi)$ are
- (A) n (B) $n\pi$
 (C) $n^2\pi^2$ (D) n^2
81. The solution of partial differential equation $xzp + yzq = xy$ is
- (A) $\phi(x | y, xy - z^2) = 0$ (B) $\phi(xy, yz - y^2) = 0$
 (C) $\phi(x | y, z + y^2) = 0$ (D) None of these.

82. The partial differential equation $(x+1)\frac{\partial^2 u}{\partial x^2} - 2(x+2)\frac{\partial^2 u}{\partial x \partial y} + (x+3)\frac{\partial^2 u}{\partial y^2} = 0$ is

- (A) Elliptic (B) Hyperbolic
(C) Parabolic (D) None of these.

83. Using Newton-Raphson method, the smallest positive root of the equation $3x^3 - 9x^2 + 8 = 0$, lying between 1 and 2, is

- (A) 1.0327 (B) 1.6514
(C) 1.2261 (D) None of these.

84. Consider the data given below :

x	0	1	2	3	4	5	6	7	8
$f(x)$	0	4.13	7.20	9.25	10.25	10.00	9.05	7.12	6.2

Using Simpson's one-third rule, the value of $\int_0^8 f(x)dx$ is

- (A) 34.5 (B) 47.3
(C) 52.8 (D) 60.4

85. Given $\frac{dy}{dx} = -y$ with $y(0) = 1$. Then using Euler's method, the value of $y(0.02)$ with step size $h = 0.01$ is

- (A) 0.5401 (B) 0.9801
(C) 2.7862 (D) 3.6845

86. The extremal of the functional $I[y(x)] = \int_1^2 (y'^2 - 2xy)dx$ subject to $y(1) = 0, y(2) = -1$ is

(A) $y = \frac{1}{6}(x - x^3)$

(B) $y = \frac{1}{6}(7x - x^3)$

(C) $y = x^2 - 1$

(D) None of these.

87. The Euler's equation corresponding to the functional $\int_0^\pi (y'^2 - y^2 + 4y \cos x)dx$ subject to $y(0) = 0, y(\pi) = 0$

(A) $\frac{d^2 y}{dx^2} + y = 2 \sin x$

(B) $\frac{d^2 y}{dx^2} + y = 2 \cos x$

(C) $\frac{d^2 y}{dx^2} - y = 2 \tan x$

(D) None of these.

88. The solution of the linear integral equation

$$\phi(x) = (1+x)^2 + \int_{-1}^1 (x\xi + x^2\xi^2)\phi(\xi)d\xi, \text{ is}$$

(A) $\phi(x) = 1 + 6x + \frac{25}{9}x^2$

(B) $\phi(x) = 1 - 6x + \frac{5}{9}x^2$

(C) $\phi(x) = 1 + 3x - \frac{25}{9}x^2$

(D) None of these.

89. The resolvent kernel for the integral equation

$$\phi(x) = 29 + 6x + \int_0^x [5 - 6(x - \xi)]\phi(\xi)d\xi, \text{ is}$$

(A) $e^{-3(x-\xi)} - 4e^{-2(x-\xi)}$

(B) $9e^{2(x-\xi)} - 4e^{3(x-\xi)}$

(C) $9e^{3(x-\xi)} - e^{-2(x-\xi)}$

(D) $9e^{3(x-\xi)} - 4e^{2(x-\xi)}$

90. Let q_i and $\dot{q}_i$ respectively are the generalized co-ordinates and velocities of a mechanical system and p_i are its generalized momenta. If H is the Hamiltonian of the system, then Hamilton's equations of motion are

(A) $\dot{q}_i = \frac{\partial H}{\partial p_i}, \dot{p}_i = \frac{\partial H}{\partial q_i}$

(B) $\dot{q}_i = -\frac{\partial H}{\partial p_i}, \dot{p}_i = \frac{\partial H}{\partial q_i}$

(C) $\dot{q}_i = \frac{\partial H}{\partial p_i}, \dot{p}_i = -\frac{\partial H}{\partial q_i}$

(D) $\dot{q}_i = -\frac{\partial H}{\partial p_i}, \dot{p}_i = -\frac{\partial H}{\partial q_i}$

91. If $\vec{L} = [L_1, L_2, L_3]$ is the vector moment of the external forces about a fixed point O , $\vec{\omega} = [\omega_1, \omega_2, \omega_3]$ the angular velocity, and A, B, C are the principal moments of inertia, then Euler's dynamical equations of motion are

(A) $L_1 = A\omega_1 - (B - C)\omega_2\omega_3, L_2 = B\omega_2 - (C - A)\omega_3\omega_1, L_3 = C\omega_3 - (A - B)\omega_1\omega_2$

(B) $L_1 = A\dot{\omega}_1 - (B - C)\omega_2\omega_3, L_2 = B\dot{\omega}_2 - (C - A)\omega_3\omega_1, L_3 = C\dot{\omega}_3 - (A - B)\omega_1\omega_2$

(C) $L_1 = A\dot{\omega}_1 + (B - C)\omega_2\omega_3, L_2 = B\dot{\omega}_2 + (C - A)\omega_3\omega_1, L_3 = C\dot{\omega}_3 + (A - B)\omega_1\omega_2$

(D) None of these.

92. Let X be a continuous random variable with probability density function (p.d.f.) defined as

$$f(x) = 6x(1-x), 0 \leq x \leq 1.$$

Then the value of number b such that $P(X < b) = P(X > b)$ is

(A) $1/2$

(B) $1/4$

(C) $3/4$

(D) $1/3$.

93. If X and Y are two random variables having joint density function given by

$$f(x, y) = \begin{cases} 6x^2y; & 0 < x < 1, 0 < y < 1 \\ 0 & ; \text{otherwise} \end{cases}$$

Then $P(X + Y < 1)$ is

(A) $1/4$

(B) $3/8$

(C) $1/10$

(D) $2/5$.

94. For a certain normal distribution, the first moment about 10 is 40 and the fourth moment about 50 is 48. Then the standard deviation of the distribution is
- (A) 4 (B) 2
(C) 3 (D) 5.
95. The sum of all the elements of any row of the transition probability matrix of a Markov Chain is
- (A) 0 (B) 2
(C) 1 (D) 4.
96. In a trivariate distribution : $\sigma_1 = 2$, $\sigma_2 = \sigma_3 = 3$, $r_{12} = 0.7$, $r_{23} = r_{13} = 0.5$. The coefficient of multiple correlation $R_{1.23}$ is
- (A) 0.1423 (B) 0.7211
(C) 0.4892 (D) 0.2155.
97. The total number of all possible Latin square of order 3 is
- (A) 12 (B) 9
(C) 6 (D) 24.
98. The mean time to failure (MTTF) for an exponential distribution with parameter λ is
- (A) λ^2 (B) λ
(C) $\frac{1}{\lambda}$ (D) $\frac{1}{\lambda^2}$.

99. The maximum value of $Z = 2x + 3y$ subject to the constraints :

$$x + y \leq 30; \quad 3 \leq y \leq 12; \quad x - y \geq 0; \quad 0 \leq x \leq 20 \text{ is}$$

(A) 32

(B) 49

(C) 60

(D) 72.

100. If $X_1, X_2, X_3, \dots, X_n$ is a random sample from a normal population $N(\mu, 1)$. Then $t = \frac{1}{n} \sum_{i=1}^n X_i^2$ is an unbiased estimator of

(A) μ^2

(B) $\mu^2 + 1$

(C) $\mu^2 - 1$

(D) $1 - \mu^2$.

ROUGH WORK

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