Truss/ Frame: A pin jointed frame is a structure made of slender (cross-sectional dimensions quite small compared to length) members pin connected at ends and capable of taking load at joints.

Such frames are used as roof trusses to support sloping roofs and as bridge trusses to support deck.

Plane frame: A frame in which all members lie in a single plane is called plane frame. They are designed to resist the forces acting in the plane of frame. Roof trusses and bridge trusses are the example of plane frames.

Space frame: If all the members of frame do not lie in a single plane, they are called as space frame. Tripod, transmission towers are the examples of space frames.

Perfect frame: A pin jointed frame which has got just sufficient number of members to resist the loads without undergoing appreciable deformation in shape is called a perfect frame. Triangular frame is the simplest perfect frame and it has 03 joints and 03 members.

It may be observed that to increase one joint in a perfect frame, two more members are required. Hence, the following expression may be written as the relationship between number of joint j, and the number of members m in a perfect frame.

$m = 2j - 3$

(a) When LHS = RHS, Perfect frame.
(b) When LHS<RHS, Deficient frame.
(c) When LHS>RHS, Redundant frame.

Assumptions

The following assumptions are made in the analysis of pin jointed trusses:

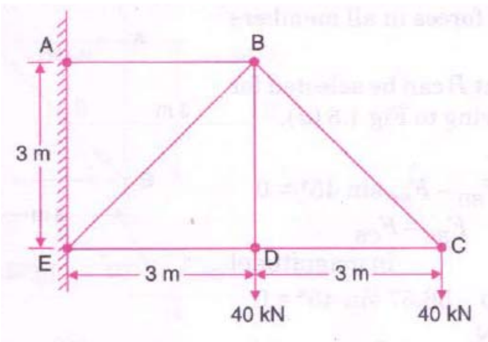
1. The ends of the members are pin jointed (hinged).
2. The loads act only at the joints.
3. Self weight of the members is negligible.

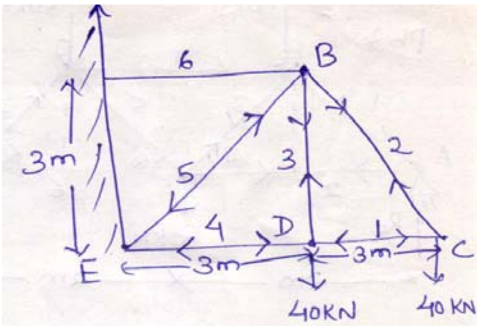
Methods of analysis

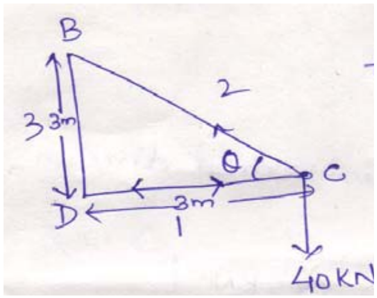
1. Method of joint
2. Method of section

Problems on method of joints

Problem 1: Find the forces in all the members of the truss shown in figure.



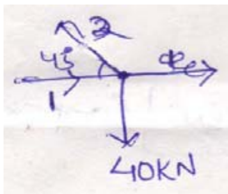




$\tan\theta = 1$

$\Rightarrow \theta = 45^\circ$

Joint C



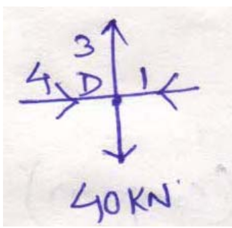
$S_1 = S_2 \cos 45$

$\Rightarrow S_1 = 40KN$ (Compression)

$S_2 \sin 45 = 40$

$\Rightarrow S_2 = 56.56KN$ (Tension)

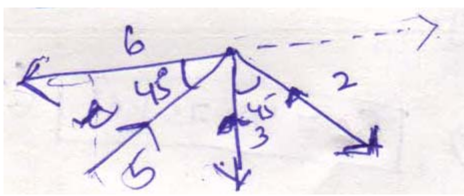
Joint D



$S_3 = 40KN$ (Tension)

$S_1 = S_4 = 40KN$ (Compression)

Joint B



Resolving vertically,

$\sum V = 0$

$S_5 \sin 45 = S_3 + S_2 \sin 45$

$\Rightarrow S_5 = 113.137KN$ (Compression)

Resolving horizontally,

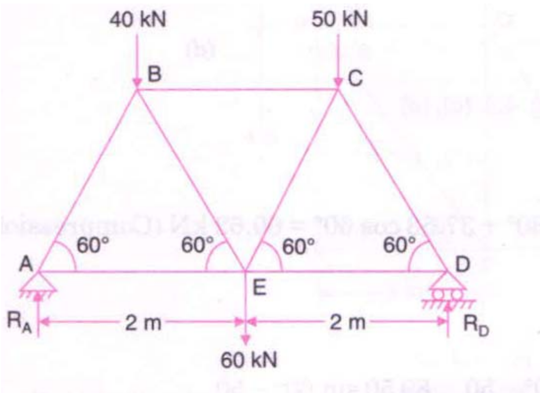
$\sum H = 0$

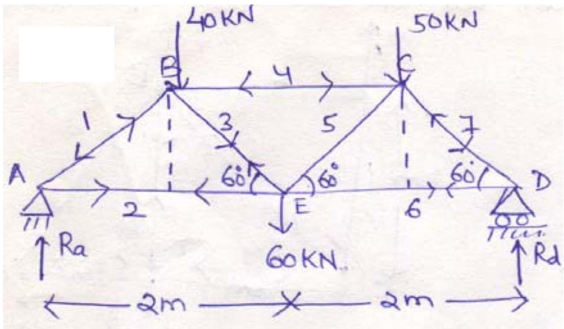
$S_6 = S_5 \cos 45 + S_2 \cos 45$

$\Rightarrow S_6 = 113.137 \cos 45 + 56.56 \cos 45$

$\Rightarrow S_6 = 120KN$ (Tension)

Problem 2: Determine the forces in all the members of the truss shown in figure and indicate the magnitude and nature of the forces on the diagram of the truss. All inclined members are at 60° to horizontal and length of each member is 2m.





Taking moment at point A,

$\sum M_A = 0$

$R_d \times 4 = 40 \times 1 + 60 \times 2 + 50 \times 3$

$\Rightarrow R_d = 77.5KN$

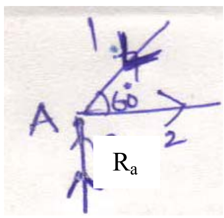
Now resolving all the forces in vertical direction,

$\sum V = 0$

$R_a + R_d = 40 + 60 + 50$

$\Rightarrow R_a = 72.5KN$

<u>Joint A</u>



$\sum V = 0$

$\Rightarrow R_a = S_1 \sin 60$

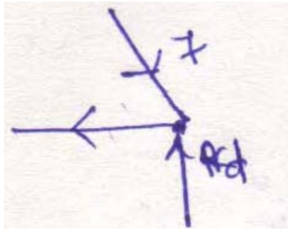
$\Rightarrow S_1 = 83.72KN$ (Compression)

$\sum H = 0$

$\Rightarrow S_2 = S_1 \cos 60$

$\Rightarrow S_1 = 41.86KN$ (Tension)

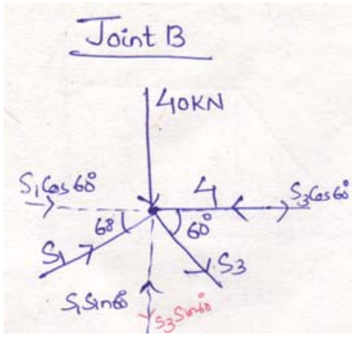
Joint D



$\sum V = 0$
$S_7 \sin 60 = 77.5$
$\Rightarrow S_7 = 89.5KN$ (Compression)

$\sum H = 0$
$S_6 = S_7 \cos 60$
$\Rightarrow S_6 = 44.75KN$ (Tension)

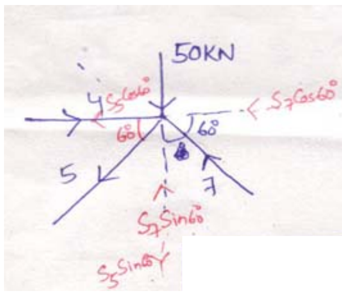
Joint B



$\sum V = 0$
$S_1 \sin 60 = S_3 \cos 60 + 40$
$\Rightarrow S_3 = 37.532KN$ (Tension)

$\sum H = 0$
$S_4 = S_1 \cos 60 + S_3 \cos 60$
$\Rightarrow S_4 = 37.532 \cos 60 + 83.72 \cos 60$
$\Rightarrow S_4 = 60.626KN$ (Compression)

Joint C



$\sum V = 0$
$S_5 \sin 60 + 50 = S_7 \sin 60$
$\Rightarrow S_5 = 31.76KN$ (Tension)

Plane Truss (Method of Section)

In case of analysing a plane truss, using method of section, after determing the support reactions a section line is drawn passing through not more than three members in which forces are unknown, such that the entire frame is cut into two separate parts.

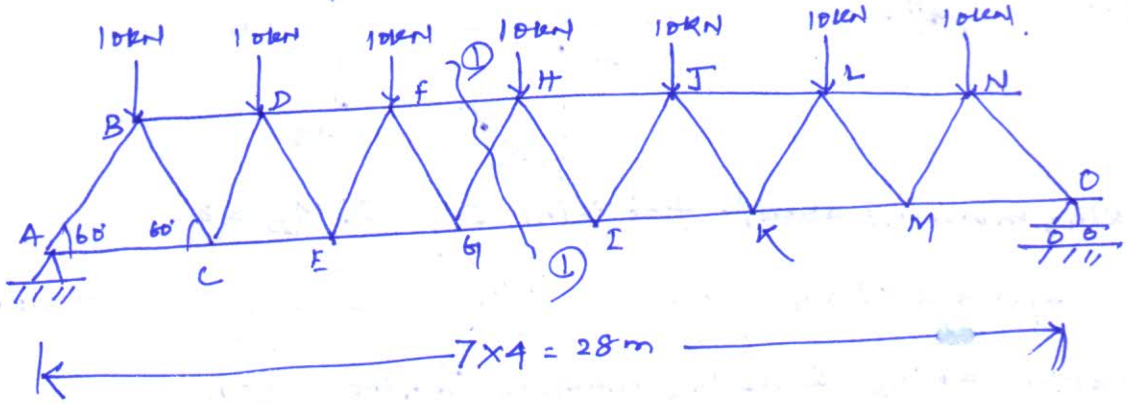
~~Ea~~ Each part should be in equilibrium under the action of loads, reactions and the forces in the members.

Method of section is preferred for the following cases:

(i) analysis of large truss in which forces in only few members are required

(ii) If method of joint fails to start or proceed with analysis for not getting a joint with only two unknown forces.

Example 1.

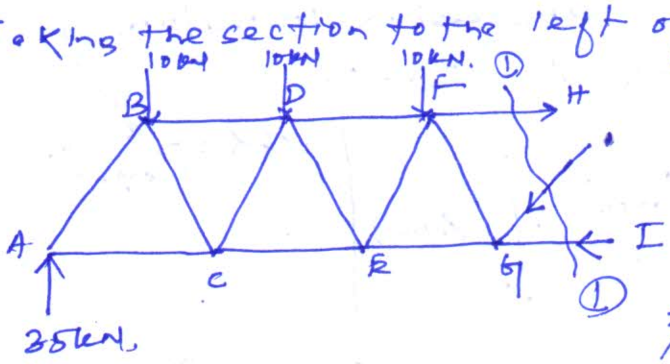


Determine the forces in the members FH, HG, and GI in the truss

Due to symmetry $R_A = R_O = \frac{1}{2} \times$ total downward load

$$= \frac{1}{2} \times 70 = \boxed{35\,KN.}$$

Taking the section to the left of the cut.



Taking moment about G

$\Sigma M_G = 0.$

$F_{FH} \times 4 \sin 60 + 35 \times 12$

$= 10 \times 2 + 10 \times 6 + 10 \times 10$

$\Rightarrow F_{FH} = \dfrac{(20+60+100) - 420}{4 \sin 60}$

$= -69.28$ kN.

Negative sign indicates that direction should have opposite i.e it is compressive in nature.

Now Resolving all the forces vertically $\Sigma Y = 0$

$$10 + 10 + 10 + F_{GH} \sin 60 = 35$$

$$\Rightarrow F_{GH} = \frac{35 - 30}{\sin 60^\circ}$$

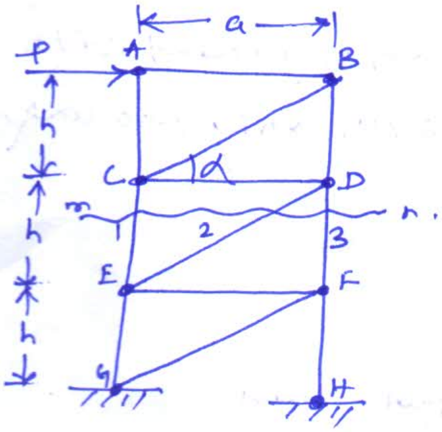
$$\Rightarrow \boxed{F_{GH} = 5.78 \text{ kN.}} \text{ (compressive)}$$

Resolving all the forces horizontally $\Sigma X = 0$.

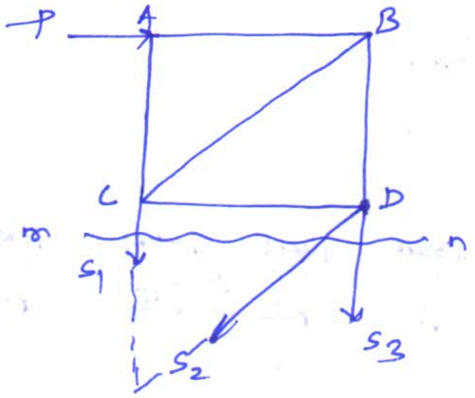
$$F_{FH} + F_{GH} \cos 60 = F_{GI}$$

$$\Rightarrow F_{GI} = 69.28 + 5.78 \cos 60^\circ = \boxed{72.17 \text{ kN.}} \text{ (tension)}$$

Q.2



Using method of sections determine the axial forces in bars 1, 2 and 3.



Taking moment about ~~poi~~ joint D $\Sigma M_D = 0$.

$$s_1 \times a = P \times h \Rightarrow \boxed{s_1 = \frac{Ph}{a}} \quad \text{—— (1)} \quad \text{(tension)}$$

Similarly taking E as the moment centre $\Sigma M_E = 0$

$$s_3 \times a + P \times 2h = 0$$

$$\Rightarrow \boxed{s_3 = \frac{-2Ph}{a}}$$

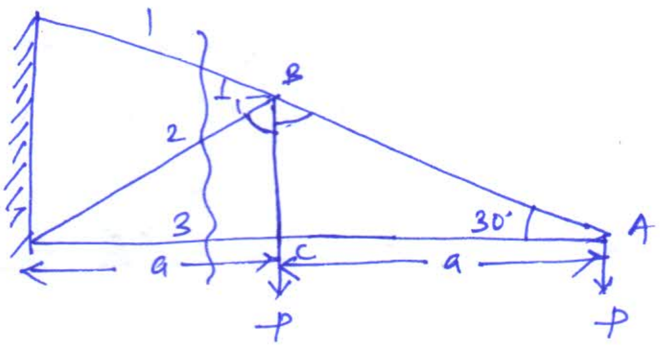
(-ve sign indicates direction of force will be opposite and it will be compressive in nature).

Resolving all the forces horizontally. $\Sigma X = 0$.

$$s_2 \cos\alpha = P$$

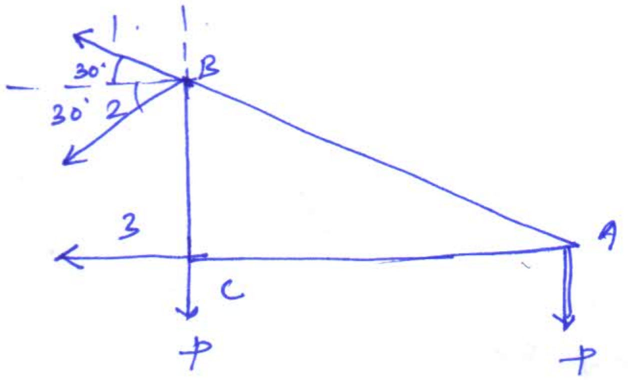
$$\Rightarrow s_2 = \frac{P}{\cos\alpha} = \boxed{\frac{P\sqrt{a^2+h^2}}{a}} \text{ (Ans)}$$

$$\cos\alpha = \frac{a}{\sqrt{a^2+h^2}}$$



$\frac{BC}{AC} = \tan 30°$

$\Rightarrow BC = a \tan 30 = \boxed{0.578\,a}$



$\Sigma M_B = 0.$

$S_3 \times 0.578\,a + P \times a = 0$

$\Rightarrow S_3 = \frac{-Pa}{0.578a} = -1.73P$

(-ve sign indicates direction is opposite and it is compressive in nature)

Resolving vertically $\Sigma Y = 0$

$S_1 \sin 30 = 2P + S_2 \sin 30$

$\Rightarrow S_1 = \frac{2P + S_2/2}{\sin 30} = (4P + S_2) \quad \text{---} (2)$

Now resolving horizontally $\Sigma X = 0$.

$S_1 \cos 30 + S_2 \cos 30 = 1.73P$

$\Rightarrow (4P + S_2) \times \frac{\sqrt{3}}{2} + S_2 \frac{\sqrt{3}}{2} = 1.73P$

$\Rightarrow 2\sqrt{3}P + \frac{\sqrt{3}}{2} S_2 + \frac{\sqrt{3}}{2} S_2 = 1.73P$

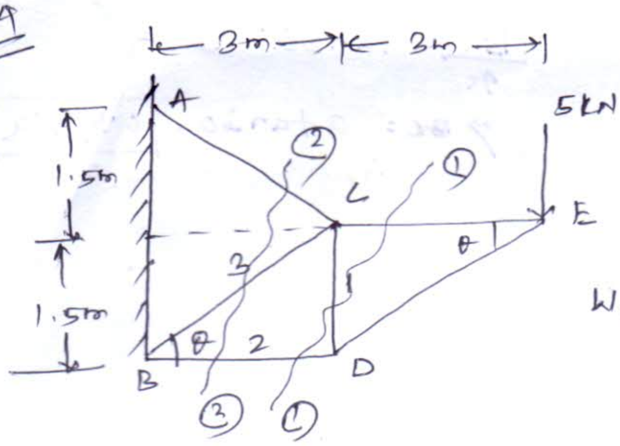
$\Rightarrow \frac{\sqrt{3}}{2} S_2 = 1.73P - 2\sqrt{3}P$

$= -1.73P$

$\Rightarrow S_2 = \frac{-1.73P}{\sqrt{3}} = \boxed{-P}$ (-ve sign indicates the direction is opposite and it is compressive)

Now $S_1 = 4P - P = \boxed{3P}$ (tension)

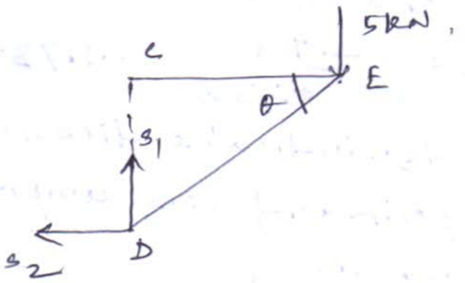
Q.4



Using method of sections, find axial forces in each bar 1, 2 and 3 of the plane truss.

We have $\tan\theta = \left(\frac{1.5}{3}\right) \Rightarrow \boxed{\theta = 26.56^\circ}$

considering section 1-1



Resolving vertically, $\Sigma Y = 0$

$\boxed{s_1 = 5kN}$ (tension)

Now taking moment about C

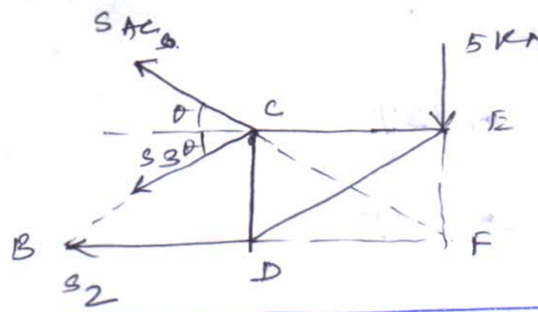
$s_2 \times 1.5 + 5 \times 3 = 0$

$\Rightarrow s_2 = -10kN$

-ve sign indicates direction should have been opposite

$\boxed{s_2 = 10kN}$ (compression)

Considering section 2-2

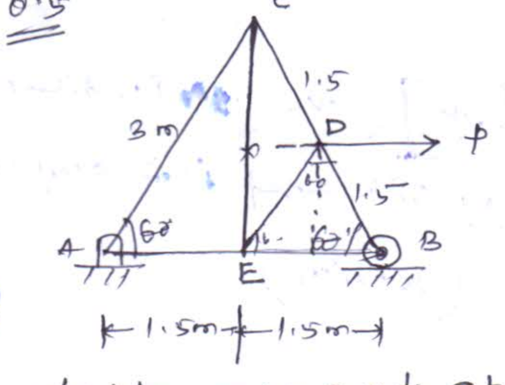


Taking moment about F

$\Sigma M_F = 0$

$\Rightarrow \boxed{s_3 = 0}$

Q.5



Assignment

Using method of joint and method of section find the axial force in the bar x.

Method of Joint

considering the whole structure and taking moment about A $\Sigma M_A = 0$.

$R_B \times 3 = P \times 1.5 \sin 60$

$\Rightarrow \boxed{R_B = \frac{\sqrt{3}}{4} P}$